Solutions to Problem 1.

$$\mathbf{P} = \begin{bmatrix} 0.5 & 0.2 & 0 & 0.3 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0.4 & 0.2 & 0.4 \end{bmatrix}$$

Solutions to Problem 2.

a. In this case, the initial state probabilities are

$$\mathbf{q} = \begin{bmatrix} 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \end{bmatrix}$$

We want $\Pr\{S_3 = 4\} = q_4^{(3)}$ = 4th element of $\mathbf{q}^{(3)} = \mathbf{q}^\top \mathbf{P}^3$.

$$\mathbf{q}^T \mathbf{P}^3 = \begin{bmatrix} 0.0328 & 0.0172 & 0.37975 & 0.19525 & 0.375 \end{bmatrix}$$

Therefore, $\Pr\{S_3 = 4\} = 0.19525$.

b. Let $\mathcal{R} = \{2, 3\}$. So,

$$\mathbf{P}_{\mathcal{R}\mathcal{R}} = \begin{bmatrix} 0.30 & 0 \\ 0 & 0.75 \end{bmatrix}$$

Looking at $\mathbf{P}_{\mathcal{R}\mathcal{R}}$, we see that $\mathcal{R} = \{2, 3\}$ does not form a self-contained Markov chain since the rows of $\mathbf{P}_{\mathcal{R}\mathcal{R}}$ do not sum to 1. Therefore, $\mathcal{R} = \{2, 3\}$ is not a recurrent class.

c. Let $\mathcal{R} = \{3, 4\}$. We want π_4.

$$\begin{aligned} \pi_{\mathcal{R}} \mathbf{P}_{\mathcal{R}\mathcal{R}} &= \pi_{\mathcal{R}} \\ \pi_{\mathcal{R}} \mathbf{1} &= 1 \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} 0.75\pi_3 + 0.50\pi_4 &= \pi_3 \\ 0.25\pi_3 + 0.50\pi_4 &= \pi_4 \\ \pi_3 + \pi_4 &= 1 \end{aligned} \quad \Rightarrow \quad \pi_3 = \frac{2}{3},\ \pi_4 = \frac{1}{3}$$

So, $\pi_4 = \frac{1}{3}$.

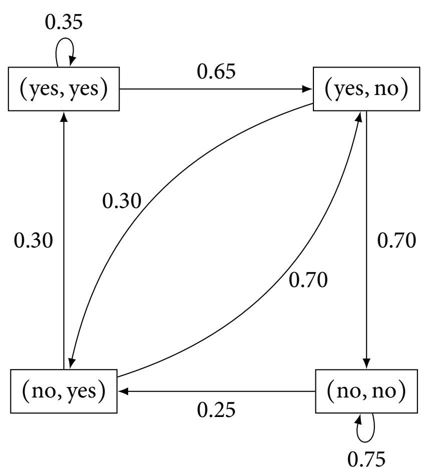
d. The set of all transient states is $\mathcal{T} = \{1, 2\}$. Let $\mathcal{R} = \{5\}$. Note that 5 is an absorbing state. We want α_{15}.

$$\alpha_{\mathcal{T}\mathcal{R}} = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.1 \\ 0.2 & 0.3 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

So, $\alpha_{15} = 0.5$.

Solutions to Problem 3.

- State space: $\{(\text{yes}, \text{yes}), (\text{yes}, \text{no}), (\text{no}, \text{yes}), (\text{no}, \text{no})\}$
  - state $(i_1, i_2) \leftrightarrow \big(\text{make } (n-1)\text{th shot?, make } n\text{th shot?}\big)$
- Time step: 1 shot
- One-step transition probabilities:



Solutions to Problem 4.

- Markov property: the probability an officer is promoted, separated, or retired next year only depends on the officer's rank this year. For example, it does not matter how long the officer has held his or her current or previous ranks.
- Time-stationarity: the probability an officer is promoted, separated, or retired next year given the officer's rank this year stays constant from year to year.